Paper - II

Mathematical Sciences

Booklet Code

A

Test Booklet No.

SUBJECT CODE : 0 4

Roll No. :

(Figures as per admission card)

Roll No. (in words) :

OMR Sheet No. :

Name and Signature of Invigilator/s

Signature :

Name :

Time : 2 Hours | Maximum Marks : 200

Number of Pages in this Booklet : 40 | **Number of Questions in this Booklet : 175**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of one hundred seventy five (175) multiple-choice type of questions. These questions are divided as Section-A (25 MCQ), Section-B (75 MCQ) and Section-C (75 MCQ). Section-A is **compulsory**. Section-B (Mathematics) and Section-C (Statistics) are optional. Candidate must choose either Section-B (Mathematics) or Section-C (Statistics) as per his/her discipline and mark it correctly on OMR answer sheet at appropriate box. Questions attempted from both the Section-B and Section-C will not be evaluated.
3. At the commencement of examination, the test booklet will be given to you. In the first 5 minutes, you are requested **To Open the Booklet and Compulsorily Examine it as Below** :
   (i) To have access to the Test Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Test Booklet will be replaced nor any extra time will be given.**
   (iii) After the verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : Ⓐ Ⓑ ● Ⓓ
   where (C) is the correct response.
5. Your responses to the questions are to be indicated in the **OMR Sheet kept inside this Booklet**. If you mark at any place other than in the circles, the OMR Sheet will not be evaluated.
6. Read the instructions given in OMR Sheet carefully. Fill the Booklet Code of Paper-II in OMR Sheet **Compulsorily**.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space provided for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the OMR answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away test booklet and carbon copy of OMR Answer Sheet after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator, electronic gadgets or log table, etc. is prohibited.**
13. **There is no negative mark for incorrect answer.**

Note : This paper contains 175 multiple choice questions of 2 marks each, in THREE (3) Sections. Attempt all the questions either from Section - A and Section - B only OR from Section - A and Section - C only. The OMR sheet with questions attempted from both Sections viz. Section - B and Section - C will not be evaluated. Number of questions Section wise:

Section - A : Q.No. 1 to 25; Section - B : Q. No. 26 to 100; Section C : Q.No. 101 to 175.

Section - A (1 - 25)

1. The sequence $\left\{x_n = \frac{n^3+3n^2}{n+1} - n^2\right\}$ is :
   (A) bounded above but not convergent
   (B) bounded below and convergent
   (C) strictly increasing and convergent
   (D) unbounded

2. For which value of $x \in \mathbb{R}$, the series $\sum_{k=1}^{\infty}\left(\frac{\cos kx}{k^3} + 3\frac{\sin kx}{k^2}\right)$ converges?
   (A) Only for when x is a rational number
   (B) Only if $x \in (0, \infty)$
   (C) Only if $x \in (-\infty, 0)$
   (D) For all $x \in \mathbb{R}$

3. The series $\sum_{n=0}^{\infty} \frac{(\lfloor n)^2}{\lfloor 2n} x^n$, $x \in \mathbb{R}$ is :
   (A) nowhere convergent
   (B) pointwise convergent but is nowhere uniformly convergent
   (C) uniformly convergent on a bounded domain
   (D) uniformly convergent only on rational numbers

4. Pointwise limit function of the sequence of functions $f_n : (-1, 1] \to \mathbb{R}$, $f_n(x) = \frac{x^n}{(1+x^n)^2}, n = 1, 2, ...$
   (A) exists and is continuous
   (B) exists, is bounded but not continuous
   (C) does not exist
   (D) exists and is differentiable

5. Let , $A_k = \left\{k + \frac{1}{2^n} \mid n \in \mathbb{N}\right\}$, where $k \in \mathbb{N}$. Let $A = (0, \infty) - \left(\underset{k \in \mathbb{N}}{\cup} A_k\right)$. Then A is :
   (A) an empty set
   (B) an uncountable set
   (C) a countably infinite set
   (D) a finite set

6. Which of the following statements is a correct statement?
   (A) There are two rational numbers whose square is 12
   (B) There are six rational numbers whose square is 12
   (C) There is exactly one rational number whose square is 12
   (D) There are no rational number whose square is 12

7. Consider $\mathbb{Q}$ and $\mathbb{R}$, the used field of rational and red numbers. Then :

(A) both of them are complete ordered field

(B) $\mathbb{Q}$ is a complete ordered field but $\mathbb{R}$ is not

(C) $\mathbb{R}$ is a complete ordered field but $\mathbb{Q}$ is not complete

(D) $\mathbb{R}$ is a complete ordered field and $\mathbb{Q}$ is also complete but not ordered

8. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 0 & , \text{ if } x \text{ is irrational} \\ \frac{1}{n} & , \text{ if } x = \frac{m}{n}, \gcd\{m, n\} = 1 \text{ and } x \neq 0 \\ 1 & , \text{ if } x = 0 \end{cases}$$

Then f is :

(A) continuous only at every rational point x

(B) continuous at all irrational points x

(C) discontinuous at all rational points and the discontinuity is of second kind

(D) discontinuous everywhere on $\mathbb{R}$

9. Let $S \subset \mathbb{R}$ be a connected set containing irrational numbers as well as rational numbers. Then S is :

(A) a countably infinite set

(B) always an interval

(C) necessarily a bounded set

(D) a disconnected set with two components

10. Let $X \subset \mathbb{R}$ be an infinite set and let $F \subset X$ be a finite set such that $X - F$ is compact. Then X is a :

(A) closed set but need not be bounded

(B) bounded set which need not be closed

(C) compact connected set

(D) compact set which need not be connected

11. Let $A = \left\{ x \in (0, 1) \mid \sin\frac{1}{x} = 0 \right\}$,

$B = \left\{ x \in \left[\sqrt{5}, 7\right] \mid x \text{ is an irrational number} \right\}$

$C = (4A) \cup (\sqrt{5}B)$, where for $a \in \mathbb{R}$ and $D \subseteq \mathbb{R}$, $aD = \{at \mid t \in D\}$. Then the Lebesgue measure m(C) of C is :

(A) $7 - \sqrt{5}$

(B) $7\sqrt{5} - 5$

(C) $7\sqrt{5} - 3$

(D) $4(7 - \sqrt{5})$

12. Consider the following statements :

Statement I : If f is Lebesgue integrable function on $\mathbb{R}$ such that $\int f(x)dx = 0$, then $f = 0$ a.e.

Statement II : If ψ is a non-negative real valued function on $\mathbb{R}$ with finite range such that $\{x \in \mathbb{R} \mid \psi(x) = t\}$ is Lebesgue measurable for every $t \in \mathbb{R}$, then ψ is Lebesgue measurable.

(A) Statement I is incorrect but statement II is true

(B) Both statement I and statement II are true

(C) Both statement I and statement II are incorrect

(D) Statement I is true but statement II is incorrect

13. Let $D \subset \mathbb{R}^2$ be open unit disk and let $f : D \to \mathbb{R}^3$ be differentiable everywhere on D. Satisfying $f'(x) = 0$ for each x in D. Then :

(A) $f(D)$ is an open subset

(B) cardinality of $f(D)$ is infinite countable

(C) $f(D)$ is necessarily a singleton set

(D) $f(D)$ can not be a compact set.

14. Let $(X_1, \|\cdot\|_1)$ and $(X_2, \|\cdot\|_2)$ be normed linear spaces and let $X = X_1 \times X_2$ be the usual product vector space.Which of the following does not define a norm on X ? Here $x \in X$ is of the form (x_1, x_2) ; $x_1 \in X_1$, $x_2 \in X_2$.

(A) $\|x\| = \max\{\|x_1\|_1, \|x_2\|_2\}$

(B) $\|x\| = \left(\|x_1\|_1^4 + \|x_2\|_2^4\right)^{1/4}$

(C) $\|x\| = \min\{\|x_1\|_1, \|x_2\|_2\}$

(D) $\|x\| = 3\|x_1\|_1 + 5\|x_2\|_2$

15. Consider the following statements :

I. X = [0, 1] is not a compact subset of $\mathbb{R}$ with discrete metric.

II. The intersection of a countable collection of dense open subsets of $\mathbb{R}$ with usual metric is dense in $\mathbb{R}$.

III. $\mathbb{R}$ with discrete metric is not a complete metric space.

IV. Every finite subset of $\mathbb{R}$ with usual metric is nowhere dense in $\mathbb{R}$.

Choose the correct answer from the options given below :

(A) I, III, IV only

(B) I, II, IV only

(C) I, II, III only

(D) II, III, IV only

16. u, v are two vectors in $\mathbb{R}^{\mathbf{n}}$ and they are both regarded as column vectors.

$M = vu^T$ ($n \times n$ matrix), then :

(A) M is always diagonalizable

(B) M is diagonalizable *iff* $u^T v = 0$

(C) M is not diagonalizable iff $u^T v \neq 0$

(D) M has a 2 × 2 Jordan block if $u^T v = 0$

17. V = Vector space of all 3×3 Hermitian matrices - that is matrices M s.t - entries of M complex and M^* = M. M^* = conjugate transpose of M.

W = Orthocomplement of the set

$\{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$

Dim V + Dim W equals :

(A) 9 (B) 11

(C) 13 (D) 10

18. v, w are vectors in $\mathbb{R}^{\mathbf{n}}$ such that :

$\|v + w\| = 8$

$\|v - w\| = 6$, $\|v\| = 5$,

then $\|w\|$ equals :

(A) 5 (B) 4

(C) 6 (D) 7

19. Let $\hat{n} = \left[\frac{1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}\right]^T$ (unit column vector in $\mathbb{R}^4$) and $A = I - c\hat{n}\hat{n}^T$.

Which of the following is true?

(A) A is invertible if c = 1

(B) A is invertible if c = 2 and $A^2 = I$

(C) A is not diagonalizable if $c = 1$

(D) $A^2 = A$ if $c \neq 0, 1$

20. A is a 5×5 matrix with first row $[-1, 1, 1, 0, 2]$ and det $A = 3$. The value of det (Adj A) equals :

Note : Adj A is the det. obtained by replacing each entry by its Cofactor and then taking the transpose.

(A) 121

(B) 81

(C) 96

(D) 125

21. A is a 3×3 real symmetric matrix with 3 distinct eigenvalues.

Given that $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ are two of its eigen vectors, which of the following is the third eigen vector?

(A) $\begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$

22. A is a 4×7 matrix such that $A^T \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix} = 0$. Consider the system of linear equations $Ax = \begin{bmatrix} 1 \\ -1 \\ \alpha \\ 7 \end{bmatrix}$

In order for this system to have a solution it is necessary that $\alpha =$

(A) $\frac{-23}{2}$ (B) $\frac{23}{2}$

(C) $\frac{-19}{2}$ (D) $\frac{19}{12}$

23. Suppose $-1, 2, 3$ are the eigen values of $a(3 \times 3)$ matrix M, then the determinant of $M^2 - 2M + 3I$ is :

(A) 112 (B) 108

(C) 126 (D) 114

24. Consider 3 planes in $\mathbb{R}^3$ given by :

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

written in matrix form as $Ax = b$; $A = [a_{ij}]_{1 \le i,\, j \le 3}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. It is given that Rank A = Rank $[A : \text{b}] = 2$. Then which of the following hold?

(A) The three planes intersect along a line

(B) The three planes form a prism. (infinite prism)

(C) The three planes intersect at a unique point

(D) The three planes are identical

25. Suppose the polynomial

$x^2 + y^2 + 2z^2 - 2xy + 4yz + 2kzx$

factors into product of linear polynomials (possibly with complex coefficients), then the value of k :

(A) – 4

(B) 4

(C) 2

(D) – 2

Section - B (26 - 100) (Mathematics)

26. Consider the function $f(z)=z-\frac{1}{2}\sin z$ the largest $R>0$ such that $f'(z)$ does not vanish on $\{z \in \mathbb{C} / |z| < R\}$ is :

(A) $\log(2+\sqrt{3})$

(B) $\log(\sqrt{3}+2)$

(C) $\log(5+\sqrt{3})$

(D) $\log(\sqrt{5}+3)$

27. Let $f : H^{-} \to \mathbb{C}$ be given by

$f(z)=\frac{1-e^z}{1+e^z}$; H^{-} is the left half plane

$H^{-} = \{z \in \mathbb{C} \ / \operatorname{Re} z < 0\}$ Denote $H_0^{+} = \{z \in \mathbb{C} / \operatorname{Re} z > 0\}$

Image of f is :

(A) H^{+}

(B) $H^{+} - \{1\}$

(C) $\mathbb{C} - \{0\}$

(D) $\mathbb{C}$

28. $\Gamma R = \{t / t \in \mathbb{R}, 0 \le t \le R\} \cup \{Re^{it} / 0 \le t \le \pi/4\} \cup \{(R-t)e^{i\pi/4} / 0 \le t \le R\}$

Using the contour ΓR and Cauchy's theorem applied to $f(z) = \exp(-z^2)$ the value of $I = \int_0^{\infty} \cos(x^2)dx$ and $J = \int_0^{\infty} \sin(x^2)dx$ are computed. Which of the following is correct?

(A) $I = J = \sqrt{\pi}/2$

(B) $I = J = \sqrt{\pi}/2\sqrt{2}$

(C) $I = 2J = \sqrt{\pi}/2$

(D) $2I = J = \sqrt{\pi}/\sqrt{2}$

29. The coefficient of z^8 in the power series expansion of $e^z \cos z$ with centre at origin is :

(A) 4/7! (B) 1/8!

(C) 1/7! (D) 2/7!

30. Let $D' = \{z \in \mathbb{C} / 0 < |z| < 1/1000\pi\}$

$f(z) = \left(\frac{\cot \pi z}{z^2}\right) \log(1 + \sin^3 \pi z)$:

for $z \in D'$. The origin is a :

(A) Removable singularity of $f(z)$

(B) Essential singularity of $f(z)$

(C) Pole of order 2

(D) Pole of order 3

31. Let $E = \{4\cos t + 9i \sin t / 0 \le t \le 2\pi\}$

The value of $\frac{1}{2\pi i} \int_E \frac{z^3 dz}{z^2+1}$ equals :

(A) −1

(B) 0

(C) 1

(D) 2

32. Consider the sum $f(z)$ of the power series :

$$f(z) = 1 + \frac{\alpha z}{2} + \frac{\alpha(\alpha-1)}{2!}\frac{z^2}{2^2} + \ldots \alpha \notin \mathbb{Z}$$

R is the radius of convergence of $f(z)$.

(A) $R = 1$ and $\left(1+\frac{z}{2}\right)f'(z) = 2\alpha f(z)$

(B) $R = 1$ and $(1+z)f'(z) = \alpha f(z)$

(C) $R = 2$ and $\left(1+\frac{z}{2}\right)f'(z) = \alpha f(z)$

(D) $R = 2$ and $\left(1+\frac{z}{2}\right)f'(z) = 2\alpha f(z)$

33. The residue of $\frac{log(z)}{(1+z^2)}$ at $z = i$ equals:

(The principal branch of log z is considered namely logz = ln|z| + i Arg z; $-\pi < Arg\, z < \pi$).

(A) 0

(B) $\pi/2$

(C) $\pi/4$

(D) $3\pi/4$

34. Let $f(z)$ be the Möbius map

$$f(z) = \frac{z+i}{2z-3}$$

$E_1 = \{z \in \mathbb{C} \,/\, |z| = 1\}$

$E_2 = \left\{z \in \mathbb{C} / z = \frac{3}{2} + te^{i\pi/4} : t \in \mathbb{R}\right\} - \left\{\frac{3}{2}\right\}$

$E_3 = \{z \in \mathbb{C} \,/\, 5|z-1| = |z-4|\} - \left\{\frac{3}{2}\right\}$

(A) f maps E_1 to a straight line

(B) f maps E_2 to a circle minus a point

(C) f maps E_1 to a circle

(D) f maps E_3 to a circle minus a point

35. Let $f(z)$ be complex differentiable (analytic) function defined as the sum of the power series $\sum_{n=1}^{\infty} z^n/n^2$, then which of the following is true?

(A) $(1-z)(zf'(z))' = -1$ on $|z| < 1$

(B) $(1+z)(zf'(z))' = -1$ on $|z| < 1$

(C) $(1+z)(zf'(z))' = 1$ on $|z| < 1$

(D) $(1-z)(zf'(z))' = 1$ on $|z| < 1$

36. The value of $\int_{-\infty}^{\infty} \frac{(\cos x)dx}{1+x^2}$ equals :

(A) e/π (B) $2e/\pi$

(C) π/e (D) $2\pi/e$

37. Value of Max $\{|z^2 - z^3| \,/\, |z| \le 1\}$ is :

(A) 2 (B) 3

(C) 4 (D) 6

38. Let $u(x, y) = \sin(x^2 - y^2)\cosh 2xy$. Note $\cosh t$ is the hyperbolic cosine defined by $\cosh t = \frac{1}{2}\left(e^t + e^{-t}\right)$. Given that u is harmonic with harmonic conjugate v, if $v(0, 0) = 0$ then the value of $v\left(\sqrt{\frac{\log 2}{2}}, \sqrt{\frac{\log 2}{2}}\right)$ equals :

(A) 3/4 (B) 3/2

(C) 3 (D) 6

39. Consider the function $f(z) = 1/\sin z$ defined on $\mathbb{C} - \{0, \pm\pi, \pm 2\pi, ...\}$. The power series for $f(z)$ centred at $\frac{\pi}{2} + 4i$ has radius of convergence :

(A) $\sqrt{64 + \pi^2}$

(B) $\frac{1}{2}\sqrt{64 + \pi^2}$

(C) $2\sqrt{64 + \pi^2}$

(D) $\frac{\pi}{2}$

40. For the function $\frac{1}{z}\log(1 + iz)$ defined on $\mathbb{C} - (\{o\} \cup \{it / t \in \mathbb{R}, t \geq 1\})$ which of the following is true?

(A) o is a simple pole

(B) o is a removable singularity

(C) i is a simple pole

(D) i is a removable singularity

41. Suppose in a group G, following equation holds :

$xyz = 1$

Consider the following statements.

Statement I : In above group $yzx = 1$ also holds.

Statement II : In above group $yxz = 1$ also hold.

Which of the following is true?

(A) Both statement I and statement II are correct.

(B) Statement I is correct but statement II is incorrect.

(C) Statement I is incorrect but statement II is correct.

(D) Both statement I and statement II are incorrect.

42. Consider the following statements :

Statement I : Let $G = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{Z}_3 \text{ and } ad - bc \neq 0\right\}$ be the group under multiplication induced from $\mathbb{Z}_3$.

Statement II : Let $H = \{g \in G \mid \det g = 1\}$

Which of the following is true?

(A) o(G) = 36 and o(H) = 18

(B) o(G) = 48 and o(H) = 12

(C) o(G) = 36 and o(H) = 9

(D) o(G) = 48 and o(H) = 24

43. Let G be a finite group such that $(ab)^3 = a^3b^3$ for all a, b in G. Consider the following statements :

Statement I : If $3 \nmid o(G)$, then G is abelian.

Statement II : If $3 \mid o(G)$, then G need not be abelian.

Which of the following is true?

(A) Both statement I and statement II are incorrect

(B) Both statement I and statement II are correct

(C) Statement I is incorrect but statement II is true

(D) Statement I is correct but statement II is incorrect

44. Let G_1 and G_2 be groups and let $f : G_1 \to G_2$ be a group homomorphism. Consider the following statements :

Statement I : If H is a normal subgroup of G_1 then f(H) is a normal subgroup of G_2.

Statement II : If H is a normal subgroup of G_2 then $f^{-1}(H)$ is a normal subgroup of G_1.

Which of the following is true?

(A) Both statement I and statement II are true

(B) Both statement I and statement II are false

(C) Statement I is correct but statement II is false

(D) Statement I is false but statement II is correct

45. Consider the subgroup $H = \{I, (12)(34), (13)(24), (14)(23)\}$ of the alternating group A_4. Then the set $A\ ut(H)$ of all automorphisms on H has :

(A) 2 elements

(B) 3 elements

(C) 6 elements

(D) 12 elements

46. The number of *Sylow* 5-subgroups of S_5 is :

(A) 1

(B) 6

(C) 12

(D) 21

47. Let p be a prime number and let $G = GL(n, \mathbb{Z}_p)$ be the general linear group of order n over the finite field $\mathbb{Z}_p$. Then centre of G is cyclic group of order :

(A) 1

(B) p - 1

(C) n

(D) p

48. Consider the following statements :

Statement I : All the non-zero elements of an integral domain with unit element have the same additive order.

Statement II : All the non-zero elements of an integral domain with unit element have the same multiplicative order.

Choose the correct option :

(A) Both statement I and statement II are incorrect

(B) Statement I is correct but statement II is incorrect

(C) Statement I is incorrect but statement II is correct

(D) Both statement I and statement II are correct

49. Consider the following statements :

Statement I : Let $A = \mathbb{R}[x]$ and let $S = \{f \in \mathbb{R}[x] \mid f(0) \in \mathbb{Z}\}$. Then S is an ideal of A.

Statement II : Let $A = \mathbb{R}[x]$ and let $P = \{f(x) \in A \mid f(1) = 0\}$. Then P is a prime ideal of A.

Choose the correct option :

(A) Both statement I and statement II are true

(B) Both statement I and statement II are false

(C) Statement I is true but statement II is false

(D) Statement I is false but statement II is true

50. Let $I = \{f(x) \in \mathbb{Z}[x] \mid f(0) = 0\}$. Then I is :

(A) a maximal ideal of $\mathbb{Z}[x]$

(B) a prime ideal but not a maximal ideal of $\mathbb{Z}[x]$

(C) an ideal of $\mathbb{Z}[x]$ but not a prime ideal

(D) not an ideal of $\mathbb{Z}[x]$

51. Which of the following statements is correct?

(A) Every subring of a PID is a PID.

(B) Quotient of a PID is a PID.

(C) The ring of all rational numbers with odd denominators in its simplest form is a PID.

(D) The cartesian product of two PIDs is a PID

52. Let I be a non-zero ideal of the ring of Gaussian integers $\mathbb{Z}[i]$. Then :

(A) $\frac{\mathbb{Z}[i]}{I}$ is an Euclidean ring

(B) $\frac{\mathbb{Z}[i]}{I}$ is a finite ring

(C) $\frac{\mathbb{Z}[i]}{I}$ is an infinite ring

(D) Depending on ideal I, $\frac{\mathbb{Z}[i]}{I}$ may be finite or infinite

53. Let K be a field of characteristic 2 and let $f(x)=x^3+3x+1\in K[x]$. If θ is one root of $f(x)$, then the other roots are :

(A) $\theta^3,\ \theta+\theta^3$

(B) $\theta^2,\ \theta+\theta^2$

(C) $1+\theta^3,\ \theta^2$

(D) $1+\theta^2,\ \theta^3$

54. Let p be a prime number and n a positive integer. Then order of the Galois group of the polynomial $x^{p^n}-1$ over the finite field $\mathbb{F}_p$. is :

(A) 1

(B) p

(C) n

(D) $p-1$

55. Let K be an extension of degree 120 of the finite field $\mathbb{Z}_2$ and let G be the group of all automorphisms of K. Then the number of distinct subgroups of G is :

(A) 2

(B) 4

(C) 8

(D) 16

56. Consider the following two statements:

Statement I : Let $L_n=\left\{\left(x,\frac{x}{n}\right)\middle|\ 0\le x\le 1\right\}$, $n\in\mathbb{N}$ be subspace of $\mathbb{R}^2$, where $\mathbb{R}^2$ is equipped with usual metric. Then $\bigcup_{n=1}^{\infty}L_n$ is connected.

Statement II : Let $\{A_n\}$ be a countable family of connected subsets in a topological space X such that $A_n\cap A_{n+1}\neq Q$ for all n in $\mathbb{N}$. Then $\bigcup_{n=1}^{\infty}A_n$ is connected.

Choose correct option from below :

(A) Both statement I and statement II are correct

(B) Statement I is true but statement II is false

(C) Statement I is incorrect and statement II is true

(D) Both statement I and statement II are incorrect

57. E is a non-empty perfect subset of $\mathbb{R}$. Then :

(A) E is necessarily connected but not necessarily compact

(B) E is necessarily compact but not necessarily connected

(C) If E is bounded, then E is necessarily compact

(D) If E is bounded, then $E\cap\mathbb{Q}\neq\phi$

58. Consider the following statements :

Statement I : Let X be a topological space which is metrizable. If X is complete and bounded then X is compact.

Statement II : Let the set $\mathbb{R}$ of real numbers be given discrete topology T_d. Then the space $(\mathbb{R}, T_d)$ is separable.

Choose the correct options.

(A) Both statement I and statement II are incorrect

(B) Statement I is correct but statement II is false

(C) Statement I is false but statement I is correct

(D) Both statement I and statement II are correct

59. Let $\mathbb{R}$ denotes $\mathbb{R}$ with usual topology, $\mathbb{R}_d$ denotes $\mathbb{R}$ with discrete topology and $\mathbb{R}^2$ denotes $\mathbb{R}\times\mathbb{R}$ with usual topology. Consider the following statements :

Statement I : The dictionary order topology on the set $\mathbb{R}\times\mathbb{R}$ is the same as the product topology $\mathbb{R}_d \times\mathbb{R}$.

Statement II : The product topology on $\mathbb{R}_d \times\mathbb{R}$ is finer than the usual topology on $\mathbb{R}^2$.

Choose correct option :

(A) Both statement I and statement II are incorrect

(B) Statement I is correct but statement II is false

(C) Statement I is incorrect but statement II is correct

(D) Both statement I and statement II are correct

60. Consider the following statements :

Statement I : The set $C(\mathbb{R}, \mathbb{Z})$ of all continuous integer valued functions, where $\mathbb{R}$ and $\mathbb{Z}$ have usual topologies, is a countable set.

Statement II : A compact connected subset of the usual space $\mathbb{R}$ of real numbers has to be necessarily a closed and bounded interval.

Choose the correct option :

(A) Both statement I and statement II are incorrect

(B) Statement I is correct but statement II is false

(C) Statement I is incorrect but statement II is true

(D) Both statement I and statement II are correct

61. Let $\mathbb{R}_\ell$ denotes $\mathbb{R}$ with lower limit topology, $\mathbb{R}_f$ denotes $\mathbb{R}$ with cofinite topology and $K=\left\{\frac{1}{n} \mid n\in\mathbb{N}\right\}$.

Which of the following is correct?

(A) $C\ell\, K = K$ in $\mathbb{R}_\ell$

(B) $C\ell\, K = \mathbb{Q}$ in $\mathbb{R}_f$

(C) $C\ell\, K = [0,1]$ in $\mathbb{R}_\ell$

(D) $C\ell\, K = \mathbb{R}$ in $\mathbb{R}_f$

62. Which of the following is incorrect? (All the spaces are equipped with usual topologies)?

(A) D^2, the closed unit disk in $\mathbb{R}^2$ and the rectangle $[0, 1] \times [0, 2]$ in $\mathbb{R}^2$ are homeomorphic

(B) Unit circle S^1 is continuous image of punctured plane $\mathbb{R}^2 - \{(0, 0)\}$

(C) The quotient space $\mathbb{R}/\mathbb{Z}$ and unit sphere S^2 are homeomorphic

(D) $S^2 - \{(0, 0, 1)\}$ and $\mathbb{R}^2$ are homeomorphic

63. Let $X_n = \{0, 1\}$ with discrete topology, $n \in \mathbb{Z}$. Let $X = \prod_{n \in \mathbb{Z}} X_n$ with product topology. Then X is a :

(A) compact connected space

(B) compact Hausdorff space

(C) compact space but not normal

(D) normal space which is not compact

64. Consider the product space X in previous question. Define $\sigma : X \to X$ by $\sigma(x_n)_{n \in \mathbb{Z}} = (y_n)_{n \in \mathbb{Z}}$ where $y_n = x_{n-1}$, $n \in \mathbb{Z}$. Then σ is :

(A) continuous one-one but not onto

(B) not continuous

(C) a homeomorphism

(D) continuous onto but not one-one

65. Which of the following is incorrect? (All the spaces have usual topologies)

(A) Every continuous function $f : D^2 \to [0, 1]$ can be continuously extended over $\mathbb{R}^2$, where D^2 is closed unit disc

(B) Disjoint closed sets in $\mathbb{R}$ can be separated by continuous functions from $\mathbb{R}$ into $[0, 1]$

(C) Let C_1 and C_2 be circles in $\mathbb{R}^2$ of radius 1 and 2. Then they can be separated by a continuous function from $\mathbb{R}^2$ into $[0, 1]$

(D) Let $f : (0, 1) \to [0, 1]$ be continuous. Then there is a continuous function $g : [0, 1] \to [0, 1]$ such that $g / (0, 1) = f$

66. Consider the inhomogeneous equation

$$y'' + P(x)y' + Q(x)y = R(x)$$

Given that $1 + e^{x^2}$, $1 + xe^{x^2}$, $(1 + x)e^{x^2} - 1$ are three solutions of the differential equation. Then, the solution with initial conditions $y(0) = 0$, $y'(0) = 0$ is :

(A) 0

(B) $1 - e^{x^2}$

(C) $x(1 - e^{x^2})$

(D) $1 + x - e^{x^2}$

67. Consider the differential equation :

$$y'' + y = \sec x, \quad \frac{-\pi}{2} < x < \frac{\pi}{2}$$

Given initial conditions

$y(0) = 0, \quad y'(0) = 0$

The value $y(\pi/3)$ equals :

(A) $\frac{\pi}{3}\frac{\sqrt{3}}{2} - \frac{1}{2}\log\frac{1}{2}$

(B) $\frac{\pi}{6}\frac{\sqrt{3}}{2} + \frac{1}{2}\log\frac{1}{2}$

(C) $\frac{\pi}{3}\frac{\sqrt{3}}{2} + \frac{1}{2}\log\frac{1}{2}$

(D) $\frac{\pi}{6}\frac{\sqrt{3}}{2} - \frac{1}{2}\log\frac{1}{2}$

68. Consider the inhomogeneous differential equation

$y'' - 2y' + y = xe^x$

Which of the following is the form of the particular solution to be used in the method of undetermined coefficients?

(A) $c_1e^x + c_2xe^x$

(B) $c_1xe^x + c_2x^2e^x$

(C) $c_1e^x + c_2x^2e^x$

(D) $c_1x^2e^x + c_2x^3e^x$

69. Wronskian of two solutions of $(1-x^2)y'' - xy' + p^2y = 0$ is denoted by $w(x)$. Given that $w(0) = 1$

What is the value of $w\left(\frac{1}{2}\right)$?

(A) $\sqrt{2}$ (B) $\sqrt{\frac{1}{2}}$

(C) $2/\sqrt{3}$

(D) $\frac{\sqrt{3}}{2}$

70. $y(x)$ is the solution of

$x^2y'' - xy' - 3y = 0$

satisfying $y(1) = 1$ and $y(x) \to 0$ as $x \to \infty$

Then value of the solution $y\left(\frac{1}{3}\right)$ is :

(A) 3 (B) $\frac{1}{3}$

(C) 27 (D) $\frac{1}{27}$

71. Given that x is a solution of

$(1-x^2)y'' + 2xy' - 2y = 0$

The second linearly independent solution $y_2(x)$ satisfies $y_2(0) = 1$, $y_2'(0) = 0$. Then $y_2\left(\frac{1}{2}\right)$ equals :

(A) $\frac{4}{5}$ (B) $\frac{5}{4}$

(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

72. Consider the following equations :

I : $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = u^2$

II : $u\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u^3$

Which of the following is true?

(A) I is semilinear and II is quasilinear

(B) II is linear and I is quasilinear

(C) I and II are both linear

(D) I and II are both quasilinear

73. Let r be the distance of a point p from the origin (in $\mathbb{R}^2$ and $\mathbb{R}^3$).

Which of the following is true?

(A) $\Delta r = 0$ in $\mathbb{R}^3$

(B) $\Delta(\log r) = 0$ in $\mathbb{R}^3 - (0)$

(C) $\Delta\left(\frac{1}{r}\right) = 0$ in $\mathbb{R}^2 - (0)$

(D) $\Delta(\log r) = 0$ in $\mathbb{R}^2 - (0)$

74. Consider the solution $u(x, t)$ of the heat equation $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0$ subject to the conditions $0 < x < \pi, \quad t > 0$

$u(0, t) = u(\pi, t) = 0$

$u(x, 0) = 8\sin^3 x$

The value of $u\left(\frac{\pi}{2}, \log 2\right)$ equals :

(A) $3 + \frac{1}{512}$ (B) $3 - \frac{1}{512}$

(C) $3 - \frac{1}{256}$ (D) $3 + \frac{1}{256}$

75. Consider the two partial differential equations :

I : $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 0$

II : $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} - 3\frac{\partial^2 u}{\partial y^2} = 0$

(A) I and II are both elliptic

(B) I and II are both hyperbolic

(C) I is elliptic and II is hyperbolic

(D) I is hyperbolic and II is elliptic

76. Under the change of coordinates $\xi = y, \ \eta = 3x + y$, the second order PDE

$u_{xx} - 6u_{xy} + 9u_{yy} = 0$

assumes the form :

(A) $w_{\xi\eta} = 0$

(B) $w_{\xi\xi} = 0$

(C) $w_{\eta\eta} = 0$

(D) $w_{\eta\eta} - w_{\xi} = 0$

77. Let $u : u(x, y)$ be the solution to the Cauchy problem $xu_x - yu_y = y$, $u(1, y) = y, \quad y \in \mathbb{R}$

Then the value of $u(2, 2)$ is :

(A) 2

(B) 4

(C) 6

(D) 8

78. Let (x_n) denote the iterative sequence generated by Newton-Raphson method with x_0 as the initial guess, when applied to find the zeros of the function $f(x)$, where $f(x)=x^2-2x-3$. If $x_0=0$, then the value of x_2 is :

(A) $-\frac{3}{2}$ (B) $-\frac{21}{20}$

(C) $\frac{3}{2}$ (D) $\frac{21}{4}$

79. Consider the following system of linear equations :

$$5x_1+x_2+x_3=7$$
$$x_1+3x_2+x_3=5$$
$$x_1+2x_2+4x_3=7$$

Let $\left(\underline{x}^{(n)}\right)$ denote the sequence of iterates generated by Gauss - Seidel method applied to the given system with $\underline{x}^{(0)}$ denoting the initial guess. Which of the following statements is true?

(A) The sequence $\left(\underline{x}^{(n)}\right)$ converges if and only if $\underline{x}^{(0)}=(1,\ 1,\ 1)$

(B) The sequence $\left(\underline{x}^{(n)}\right)$ converges if and only if $\underline{x}^{(0)}\in\{(0,0,0),(1,1,1)\}$

(C) The sequence $\left(\underline{x}^{(n)}\right)$ converges for every $\underline{x}^{(0)}\in\mathbb{R}^3$

(D) The sequence $\left(\underline{x}^{(n)}\right)$ does not converge for any choice of $\underline{x}^{(0)}$

80. The degree of precision (order of exactness) of Simpson's rule, which is used to approximate integrals, is :

(A) 1 (B) 2

(C) 3 (D) 4

81. The values of a continuous function $f(x)$ defined on the interval [0, 4] are tabulated below :

x	0	1	2	3	4
$f(x)$	1	1	2	0.5	1

The approximate value of the integral $\int_0^4 f(x)dx$ computed using composite Simpson's rule and the given data is :

(A) 4

(B) $\frac{10}{3}$

(C) $\frac{20}{3}$

(D) 8

82. Using two steps of Euler method, the value of $y(0.2)$ where $y(x)$ is the solution to the initial value problem

$$\frac{dy}{dx}=y^2,\quad y(0)=1$$

is :

(A) 1.12

(B) 1.121

(C) 1.22

(D) 1.221

83. Using one step of Runge-Kutta method of order 2, the value of $y(0.2)$ where $y(x)$ is the solution to the initial value problem

$$\frac{dy}{dx} = 3y, \quad y(0) = 1$$

is :

(A) 1.15

(B) 1.39

(C) 1.46

(D) 1.78

84. Let $f : [-3, 3] \to \mathbb{R}$ be defined by

$$f(x) = (x+2)(x+1)^2(x-2)(x-1)$$

If bisection method is used to find a zero of the function $f(x)$ starting with $[-3, 3]$ as the initial interval, then the sequence of bisection method iterates converge to :

(A) – 2

(B) – 1

(C) 1

(D) 2

85. Let $p(x)$ be the polynomial of least degree such that

$p(0) = 1$, $p'(0) = 0$, $p(1) = 2$, $p'(1) = 0$.

Then the value of $p(x)$ at $x = 2$ is :

(A) – 4

(B) – 3

(C) 3

(D) 4

86. Let $p(x)$ be the interpolating polynomial for the data

x	1	2	3	4
y	1	5	1	1

Then the value of $p(x)$ at $x = 5$ is :

(A) 17 (B) 13

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

87. The Lagrange form of the interpolating polynomial for the data

x	–1	0	2
y	3	2	6

is :

(A) $x(x-2)+(x+1)(x-2)+x(x+1)$

(B) $x(x-2)-(x+1)(x-2)+x(x+1)+36x(x+1)(x-2)$

(C) $x(x-2)-(x+1)(x-2)+x(x+1)$

(D) $x(x-2)+(x+1)(x-2)+x(x+1)+12x(x+1)(x-2)$

88. Let $y = y(x)$ be a twice continuously differentiable function defined on the interval [0, 1]. Further assume that $y = y(x)$ is the stationary function (also known as extremal) of the functional $J[y(x)] = \int_0^1 e^y (y^1)^2 dx$ satisfying the boundary conditions $y(0) = 0$, $y(1) = \ell n 4$.

Then the value of $y(x)$ at $x = 0.5$ is :

(A) ℓn 1.25 (B) ℓn 2

(C) ℓn 2.25 (D) ℓn 8

89. Which of the following statements is True regarding extremals of the functional $J[y(x)] = \int_0^1 e^{x+y} \cdot y'(x)dx$, satisfying the boundary conditions $y(0) = 0,\ y(1) = 1$?

(A) Exactly one extremal

(B) Exactly two extremals

(C) No extremals

(D) All continuously differentiable function y(x) satisfying $y(0) = 0,\ y(1) = 1$ are extremals

90. The number of stationary functions (also known as possible extremals) of the functional $J[y(x)] = \int_0^{2\pi} (y^2 - (y^1)^2)dx$ that satisfy the boundary conditions $y(0) = 1,\ y(2\pi) = 0$ is :

(A) Zero (B) One

(C) Two (D) Infinite

91. Let $c'[0, 1]$ denote the space of all functions which are continuous along with their first derivates, defined on the interval [0, 1]. Define $J : c'[0, 1] \to \mathbb{R}$ by

$$J[y(x)] = \int_0^1 \left(\frac{dy}{dx}(x)\right)^2 dx$$

Then the (strong) variation of the functional J at any function $y = y(x)$, in the usual notation, is given by :

(A) $\int_0^1 2\frac{dy}{dx}\frac{d}{dx}(\delta y)dx$ (B) $\int_0^1 2\frac{dy}{dx}\frac{d^2y}{dx^2}\delta y\, dx$

(C) $\int_0^1 2\,\delta y\, y\, dx$ (D) $\int_0^1 2\frac{dy}{dx}\frac{d^2y}{dx^2}\frac{d}{dx}(\delta y)\, dx$

92. For every pair f, g of real-valued continuous functions defined on the interval [0, 1] such that the corresponding derivatives f ', g' are also continuous on [0, 1], define

$\rho_0[f(x), g(x)] := \max_{0 \le x \le 1} |f(x) - g(x)|$, and

$\rho_1[f(x), g(x)] := \max\{\rho_0[f(x), g(x)],\ \rho_0[f'(x), g'(x)]\}$

Let $f_n(x) = \frac{\sin n^2 x}{n}$, $g(x) = 0$ for $0 \le x \le 1$

Then the sequence of $\rho_1[f_n(x), g(x)]$

(A) is bounded and converges to zero

(B) is bounded and does not converge

(C) is unbounded and has a subsequence that converges to zero

(D) diverges to $+\infty$

93. The characteristic number for which the function cos3x is an eigenfunction of the integral equation

$$\varphi(x) - \lambda\int_0^{\pi}\left(\cos^2 x\cos 4t + 4\cos 3x\cos^3 t\right)\varphi/t)dt = 0$$

is :

(A) $\frac{2}{\pi}$

(B) $\frac{4}{\pi}$

(C) $\frac{8}{\pi}$

(D) $\frac{16}{\pi}$

94. Let $\varphi(x)$ be the function which is not identically zero, and solves the integral equation

$$\varphi(x) = 2\int_0^1 xt(\varphi(t))^2\,dt$$

Then the value of $\varphi(x)$ at $x = \frac{1}{4}$ is :

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 1

95. Which of the following Volterra integral equations of second kind has the same set of solutions as the Volterra integral equation of first kind given by

$$\int_0^x \left(2 + \sin(x+t)\right)\varphi(t)\,dt = x$$

assuming that set of solutions is non-empty.

(A) $\varphi(x) = -\int_0^x \frac{\cos(x+t)}{2+\sin 2x}\varphi(t)dt - \frac{x}{2+\sin 2x}$

(B) $\varphi(x) = -\int_0^x \frac{\sin(x+t)}{2+\sin 2x}\varphi(t)dt - \frac{x}{2+\sin 2x}$

(C) $\varphi(x) = \int_0^x \frac{\sin(x+t)}{2+\sin 2x}\varphi(t)dt + \frac{1}{2+\sin 2x}$

(D) $\varphi(x) = -\int_0^x \frac{\cos(x+t)}{2+\sin 2x}\varphi(t)dt + \frac{1}{2+\sin 2x}$

96. Let $R(x, t\,;\lambda)$ be the resolvent Kernel for the integral equation

$$\varphi(x) = x + \lambda\int_0^x \frac{2+x^2}{2+t^2}\varphi(t)dt$$

The value of $R\,(2, 1\,; 1)$ is :

(A) $\frac{5}{2}e$

(B) 2e

(C) e^2

(D) $e^{5/2}$

97. Let $y(x)$ be the solution to the ordinary differential equation $\frac{d^2y}{dx^2} + xy = \sin x$ satisfying the initial conditions

$y(0) = 1, \quad \frac{dy}{dx}(0) = 0$

If $\varphi(x)$ denotes the function $\frac{d^2y}{dx^2}(x)$, then the integral equation satisfied by $\varphi(x)$ is :

(A) $\varphi(x) = \int_0^x x(x-t)\varphi(t)\,dt + x - \sin x$

(B) $\varphi(x) = \int_0^x x(x-t)\varphi(t)\,dt + \cos x - x$

(C) $\varphi(x) = -\int_0^x x(x-t)\varphi(t)\,dt + \sin x - x$

(D) $\varphi(x) = \int_0^x x(x-t)\varphi(t)\,dt + x - \cos x$

98. Consider the motion of a simple pendulum of length ℓ moving in a plane under the influence of gravity governed by $y'' + \frac{g}{\ell}\sin y = 0$. Here g is the acceleration due to gravity pointing vertically downwards. Given initial conditions $y(0) = 0$, $y'(0) = c > 0$ such that $\lim_{t\to\infty} y'(t)$ exists and $\lim_{t\to+\infty} y(t) = \pi$. The possible value of c is:

(A) $\sqrt{\frac{4g}{\ell}}$ (B) $\sqrt{\frac{2g}{\ell}}$

(C) $\sqrt{\frac{g}{\ell}}$ (D) No such $c > 0$ exists

99. Consider the motion of a particle in the plane, where $(x(t), y(t))$ is the instantaneous position of the particle at time t. Suppose the governing equations of motion are

$x'' + 4x = 0, \quad y'' + by = 0$ where b is a positive real number then the motion is periodic for the value $b =$

(A) 6 (B) 7

(C) 8 (D) 9

100. Consider the Eulerian equations of motion of a rigid body given by

$$A\dot{w}_1 = (B - C)w_2w_3$$
$$B\dot{w}_2 = (C - A)w_1w_3 \;;\; A > B > C > 0$$
$$C\dot{w}_3 = (A - B)w_1w_2$$

A, B, C are the principal moments of inertia of the body about a point P in the body. The rigid body rotates with the point P remaining fixed for all times.

The rotational Kinetic Energy is :

(A) $\frac{1}{2}\left(A(w_1(t))^2 + B(w_2(t))^2 + C(w_3(t))^2\right)$

(B) $\left(A(w_1(t))^2 + B(w_2(t))^2 + C(w_3(t))^2\right)$

(C) $\frac{1}{2}\left(A^2(w_1(t))^2 + B^2(w_2(t))^2 + C^2(w_3(t))^2\right)$

(D) $\left(A^2(w_1(t))^2 + B^2(w_2(t))^2 + C^2(w_3(t))^2\right)$

Section - C (101 - 175) (Statistics)

101. Consider the following two statements :

   P : Ogive is drawn for more than type cumulative frequencies

   Q : Ogive is drawn for less than type cumulative frequencies

   Which one of the following is correct?

   (A) *P* is true but *Q* is not
   (B) *Q* is true but *P* is not
   (C) Both *P* and *Q* are true
   (D) Neither *P* nor *Q* is true

102. If 10 is subtracted from each of the 10 observations, then which of the following is not correct?

   (A) Mode will be decreased by 10
   (B) Mean will be decreased by 10
   (C) Median will be decreased by 10
   (D) Variance will be decreased by 10

103. Which of the following is not correct?

   (A) Median is 2nd quartile
   (B) Median is 5th decile
   (C) Median is 50th percentile
   (D) Median is the arithmetic mean of 4th decile and 6th decile

104. The arithmetic mean of 11 observations is 11. If the arithmetic mean of the first 6 observations is 10.5 and that of the last 6 observations is 11.5, the value of the 6th observation is :

   (A) 10.0 (B) 10.5
   (C) 11.0 (D) 11.5

105. For comparing dispersion of two sets of positive observations which of the following should be used?

   (A) Range
   (B) Standard deviation
   (C) Mean deviation
   (D) Coefficient of variation

106. For two sets of observations consisting of some extreme values, dispersion can be compared using :

   (A) Quartile deviation / |Median|
   (B) Quartile deviation / |Mean|
   (C) Range / Median
   (D) Range / Standard deviation

107. Mean deviation is minimum when the deviations are taken from :

   (A) Zero
   (B) Mean
   (C) Median
   (D) Mode

108. For a set of 10 observations, mean deviation about mean, standard deviation, and median are 5, 8 and 12.4, respectively. Which of the following could be a value of mean deviation about the point 12.4?

   (A) 0
   (B) 3.7
   (C) 6.2
   (D) 8.5

109. If each of 5 positive observations is multiplied by 5, then the coefficient of variation :

(A) will remain same

(B) will increase by 5%

(C) will decrease by 5%

(D) may increase or decrease depending on whether the values are less than 1 or greater than 1

110. For a distribution which of the following is always correct?

(A) Median lies between mean and mode

(B) Root mean square deviation about mean is the least

(C) Mean-Mode = 3(Mean - Median)

(D) Mean = Median = Mode

111. Which of the following is correct?

(A) Standard deviation is always positive

(B) Standard deviation may take any value other than zero

(C) Mean of a distribution can take any value on the real line

(D) A leptokurtic distribution cannot be asymmetric

112. For three events A, B and C, which of the following is not correct?

(A) If $P(A)=1$ then A is an almost sure event

(B) If $P(A)=0$, then A is an impossible event

(C) $P(A|B)<\frac{P(A)}{P(B)}$ with $P(B)>0$

(D) If $P(A)>0$ and $P(B)>0$, then $P(A^c \cap B)+P(B^c \cap A)<P(A \cup B)$

113. A bag contains 4 white balls and 3 black balls. Two random draws of two balls (at each draw) are made successively. What is the probability of getting 2 white balls at the first draw and 2 black balls at the second draw when the balls drawn at the first draw are replaced back to the bag before the second draw?

(A) $\frac{1}{7}$

(B) $\frac{3}{7}$

(C) $\frac{2}{49}$

(D) $\frac{19}{49}$

114. An unbiased coin is tossed 5 times and it is observed that first 4 tosses result in heads. Which of the following is true?

(A) The fifth toss will result in a tail

(B) The fifth toss will have a tail with probability 1/5

(C) The fifth toss will result in a tail with probability less than 1/5

(D) The fifth toss will result in a tail with probability 1/2

115. In a shooting competition Arun shoots at the target 4 times out of 5 shots, Tushar shoots 5 times out of 6 and Vishnu 3 times out of 4 shots. What is the probability that the target will be hit at least once when each of them shoots once?

(A) $\frac{1}{120}$

(B) $\frac{119}{120}$

(C) $\frac{118}{120}$

(D) $\frac{46}{120}$

116. Let a usual biased die and a fair coin be tossed once together, where the face value 6 turns up with probability 1/2 and other face values are equality likely among themselves. Define $X = i$ if i appears on the die, and $Y = 1$ if head appears with the coin and $Y = 0$ otherwise. Find $E(XY)$.

(A) $\frac{9}{4}$ (B) $\frac{15}{4}$

(C) $\frac{21}{4}$ (D) $\frac{27}{4}$

117. In answering a question on a multiple choice test having 4 options with exactly one correct answer, an examine either knows the answer (with probability p) or guesses. If he guesses the answer, he does it at random from the 4 alternatives. If he answers a question correctly what is the probability that he actually knows the answer?

(A) $\frac{4p}{1+4p}$

(B) $\frac{4p}{1+3p}$

(C) $\frac{3p}{1+3p}$

(D) $\frac{3p}{1+4p}$

118. We have a set of 10 observations with the minimum value 5 and the maximum value 15. Let s be the standard deviation of the 10 observations. Then which of the following is correct?

(A) $s \in (0, 1]$

(B) $s \in (1, 3]$

(C) $s \in (3, 5]$

(D) $s \in (5, 10]$

119. Let m_i be the ith order central moment defined on 10 observations. If $b_1 = m_3^2 / m_2^2$ and $b_2 = m_4 / m_2^2$, then which of the following is not correct?

(A) $b_2 > b_1$

(B) $b_2 = b_1$ if all the observations are same

(C) $b_2 \geq b_1 + 1$

(D) $b_2 \geq 1$

120. Let Q_1, Q_2, Q_3 be the three quartiles of a set of values. Define $s_k = (Q_3 - 2Q_2 + Q_1)/(Q_3 - Q_1)$. Then which of the following is correct?

(A) $s_k \in [-3, -2]$

(B) $s_k \in [-2, -1]$

(C) $s_k \in [-1, 1]$

(D) $s_k \in [0, 2]$

121. Suppose x and y are related as $5x - 2y + 4 = 0$. Let r be the correlation coefficient between x and y. Which of the following is correct?

(A) $r = 1$

(B) $r = -1$

(C) $r = 0.6$

(D) $r = -0.46$

122. Let the variables x, y, u, v be related as $4u = 2x + 7$, $6v = 2y - 15$. Suppose the regression coefficient of y on x, b_{yx}, is given to be 3. Then what is the value of b_{vu}, the regression coefficient of v on u?

(A) 1

(B) – 1

(C) 2

(D) 0

123. For any random variable X, the median of X is the point m such that :

(A) $P(X \leq m) \geq \frac{1}{2}$ and $P(X \geq m) \leq \frac{1}{2}$

(B) $P(X \geq m) \geq \frac{1}{2}$ and $P(X \leq m) \leq \frac{1}{2}$

(C) $P(X \leq m) \geq \frac{1}{2}$ and $P(X \geq m) \geq \frac{1}{2}$

(D) $P(X < m) > \frac{1}{2}$ and $P(X > m) > \frac{1}{2}$

124. Suppose X is a continuous random variable having probability density function f with mean $\mu(<\infty)$ and median m. Then which of the following is correct?

(A) $\int_{-\infty}^{\infty}|x-A|f(x)dx$ is minimum if $x=\mu$

(B) $\int_{-\infty}^{\infty}|x-A|f(x)dx$ is minimum if $x=m$

(C) $\int_{-\infty}^{\infty}|x-\mu|f(x)dx \geq \sqrt{\frac{1}{n}\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx}$

(D) $\int_{-\infty}^{\infty}|x-m|f(x)dx \geq \sqrt{\frac{1}{n}\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx}$

125. Which of the following is correct?

(A) For a binomial distribution, mean = standard deviation

(B) Binomial distribution with success probability p is positively skewed if $p > 0.5$

(C) Binomial distribution with success probability p is leptokurtic if $p(1-p) > \frac{1}{6}$

(D) Binomial distribution is symmetric if success probability is same as failure probability

126. For two random variables X and Y, suppose $E(Y|X=x) = 2+\frac{x}{3}$ and $E(X|Y=y) = 2y-1$. Then :

(A) $E(X) = 9/2$, $E(Y) = 5/2$

(B) $E(X) = 9/2$, $E(Y) = 7/2$

(C) $E(X) = 7/2$, $E(Y) = 9/2$

(D) $E(X) = 9$, $E(Y) = 5$

127. Suppose a coin with probability of getting head as p is tossed again and again. Let X be the number of times a head appears. Then which one of the following statements is correct?

(A) The distribution of X is symmetric if $p = 1/2$

(B) If $p(>0)$ is a very small positive number, then the probability that no head appears in an infinite tosses of the coin is greater than zero

(C) The distribution of X is negatively skewed if $p < 1/2$

(D) The distribution of X is always positively skewed

128. Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables with mean μ and variance σ^2. Write $\sum_{i=1}^{n} X_i = n\bar{X}_n$. It is known that:

$$P\left(|\bar{X}_n - \mu| \geq k\sigma\right) \leq \frac{1}{nk^2} \qquad (1)$$

for any $k > 0$. Then which of the following is not correct?

(A) If $n = 1$ and $k \in (0, 1)$, then equality in (1) cannot be obtained

(B) If $n = 2$, then equality in (1) holds if and only if $k = 1$

(C) If $n = 3$, then equality in (1) cannot be obtained

(D) In order to get equality in (1), X cannot be a discrete random variable

129. Which of the following is not correct?

(A) Let X_n be a sequence of uniformly distributed random variables over the set $\left\{\frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}, 1\right\}$. Then $X_n \xrightarrow{L} X \sim U(0, 1)$

(B) If a sequence of distribution functions $\{F_n\}$ converges to F in distribution, then F must be a distribution function

(C) Convergence of sequence of distribution functions may not imply the convergence of corresponding density functions

(D) If $X_n \xrightarrow{P} X$, then $X_n \xrightarrow{L} X$

130. Let i and j be two states of a Markov chain. Define a relation iRj if i communicates with j. Then :

(A) the relation is reflexive but not symmetric

(B) the relation is symmetric but not transitive

(C) the relation is reflexive but not transitive

(D) the relation is an equivalence relation

131. Which of the following is not correct?

(A) Stationary distribution of an irreducible Markov chain is unique.

(B) Limiting distribution of a Markov chain is a stationary distribution of the chain.

(C) If a Markov chain is irreducible and aperiodic, then its limiting distribution exists.

(D) If a random walk is symmetric, then its stationary distribution exists.

132. For which value of k, $f(x) = kx^2e^{-x}$, $x > 0$ is a probability density function?

(A) 0

(B) 1/2

(C) 1

(D) 2

133. Let $f_Z(\cdot)$ be the density function of a random variable Z. if two random variables X and Y are related as $Y = g(X)$ with g strictly monotone, which one of the following is correct?

(A) $f_Y(y) = \left[f_X(x) \left| \frac{dx}{dy} \right| \right]_{x=g^{-1}(y)}$

(B) $f_Y(y) = \left[f_X(x) \left| \frac{dy}{dx} \right| \right]_{x=g^{-1}(y)}$

(C) $f_Y(y) = \left[f_X(x) \frac{dx}{dy} \right]_{x=g^{-1}(y)}$

(D) $f_Y(y) = \left[f_X(x) \frac{dy}{dx} \right]_{x=g^{-1}(y)}$

134. Let X and Y be two independent random variables with variance of X as 2 and that of Y as 3. What is the variance of $5X - 3Y$?

(A) 1

(B) 19

(C) 23

(D) 77

135. A random variable X has the moment generating function $M_X(t) = \left(\frac{1}{4} + \frac{3}{4} e^t \right)^5$. Then $E(X)$ and $V(X)$ are given respectively by :

(A) $\frac{15}{4}, \frac{15}{4}$

(B) $\frac{15}{4}, \frac{15}{16}$

(C) $\frac{1}{4}, \frac{5}{4}$

(D) $\frac{1}{4}, \frac{3}{4}$

136. Let X be a Poisson random variable such that $P(X=2) = 9P(X=4) + 90P(X=6)$. What is the expected value of X?

(A) 1

(B) 2

(C) 0

(D) 5

137. The probability of getting a success in each trial be 2/3. Then the expected number of trials required to get the first success is :

(A) 3/2

(B) 3

(C) 1/3

(D) 1/2

138. For a Poisson random variable X, if $E(X^2) = 6$, what is the value of $E(X)$?

(A) 1

(B) 2

(C) 3

(D) 4

139. A stochastic matrix A is regular if :

(A) each element of A is positive

(B) each element of A is non-negative

(C) each element of A^m is positive for all integral m

(D) each element of A^m is positive for at least one integral m

140. Let X be a two-dimensional random vector with $E(X) = (2 \quad 3)'$ and $D(X) = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$. Two observations $x_1 = (1 \quad 1)'$ and $x_2 = (3 \quad 3)'$ are observed. Find the statistical distance between x_1 and x_2 :

(A) $\sqrt{20}$

(B) $\sqrt{15}$

(C) $\sqrt{3}$

(D) 2

141. Which of the following is not correct?

(A) A random variable X can be made 'not a random variable' by changing the corresponding σ-field.

(B) Support of a random variable is a closed set.

(C) Support of a random variable is a perfect set.

(D) Support of a continuous random variable is dense in itself.

142. Let $X = (X_1, X_2, X_3)' \sim N_3(\mu, \Sigma)$ where $\mu = (1 \quad 2 \quad -3)'$ and $\Sigma = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Which one of the following is correct?

(A) X_1, X_2 are independent

(B) X_1, X_3 are independent

(C) (X_1, X_3) and X_2 are independent

(D) (X_1, X_2) and X_3 are independent

143. Let $(X, Y) \sim N_2(\mu, \Sigma)$. Then the maximum likelihood estimates of μ and Σ based on the sample (3, 6), (4, 4), (5, 7), (4, 7) is given by :

(A) $\hat{\mu} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \hat{\Sigma} = \begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 3/2 \end{pmatrix}$

(B) $\hat{\mu} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \hat{\Sigma} = \begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix}$

(C) $\hat{\mu} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \hat{\Sigma} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2 \end{pmatrix}$

(D) $\hat{\mu} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \hat{\Sigma} = \begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 3/2 \end{pmatrix}$

144. Suppose $(X, Y) \sim N_2(\mu, \Sigma)$ with $\mu = (\mu_1 \quad \mu_2)'$ and $\Sigma = ((\sigma_{ij}))$. Then which of the following is not true for the corresponding equi-probability contours?

(A) They are concentric ellipses.

(B) They are circles when the correlation coefficient between X and Y is zero.

(C) They may not be circles even if X and Y are independent

(D) They are concentric ellipses when X and Y are uncorrelated

145. Let $(X, Y) \sim N_2(\mu, I_2)$ where $\mu = (1 \quad 1)'$. Then the value of $E(X^2 + XY + 3Y^2)$ is :

(A) 9

(B) 4

(C) 5

(D) 12

146. Let $(X,Y) \sim N_2(0, I_2)$. Then the value of $V(X^2 + XY + 3Y^2)$ is :

(A) 21

(B) 43/2

(C) 9

(D) 22

147. Let $(X,Y) \sim N_2(\mu, I_2)$ where $\mu = (1 \quad 1)'$. Then which of the following is not correct?

(A) $E(X^2 + Y^2 - 2X - 2Y) = 0$

(B) $V(X^2 + Y^2 - 2X - 2Y) = 4$

(C) The random variable $X^2 + Y^2 - 2X - 2Y + 2$ has a chi-square distribution with 2 degrees of freedom

(D) $E(X^2 + Y^2 - 2X - 2Y) = 2$

148. Let $X = (X_1 \quad X_2 \quad X_3)'$ be a 3-dimensional random vector. Which of the following is not correct?

(A) If X_1, X_2, X_3 are independent and identically distributed random variables then they must be exchangeable.

(B) If X_1, X_2, X_3 are exchangeable random variables they must be independent.

(C) If X_1, X_2, X_3 are exchangeable random variables they must have identical marginals.

(D) If X_1, X_2, X_3 have identical marginals, they may not be exchangeable.

149. Suppose $X_i \sim N_p(0, \Sigma),\ i = 1, 2, \ldots, n$ independently. Define $A = \sum_{i=1}^{p} X_i X_i'$. Let us denote the distribution of A by $W_p(n, \Sigma)$. Let us partition A and Σ as :

$$A = \begin{pmatrix} A_{11}^{p_1 \times p_1} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{11}^{p_1 \times p_1} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

Write $p_2 = p - p_1$. Then which of the following is correct?

(A) $A_{11} \sim W_p(n, \Sigma_{11})$

(B) $A_{11} \sim W_{p_1}(n, \Sigma_{11})$

(C) A has $n-1$ degrees of freedom

(D) $A_{22} - A_{21}A_{11}^{-1}A_{12} \sim W_p(n, \Sigma^*)$ where $\Sigma^* = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$

150. Let multiple correlation coefficient between X_1 and $(X_2 \quad X_3 \ldots X_p)'$ be denoted by $\rho 1.23 \ldots p$. Then which one is correct?

(A) $-1 \leq \rho 1.23 \ldots p \leq 1$

(B) $0 \leq \rho 1.23 \ldots p \leq 1$

(C) $-\frac{1}{p-1} \leq \rho 1.23 \ldots p \leq 1$

(D) $\rho 1.23 \ldots p$ is increasing in p

151. Which one is not correct?

(A) In ANOVA all explanatory variables are qualitative in nature.

(B) In ANCOVA some explanatory variables are qualitative and some are quantitative in nature.

(C) In multiple regression, we have at least two explanatory variables.

(D) If response variables are quantitative in nature the corresponding regression is called logistic regression.

152. Replication in experiment means :

(A) The number of blocks

(B) Total number of treatments

(C) The number of times a treatment occurs in an experiment

(D) Systematic influences

153. Which one of the following is not correct?

(A) Experimental error is due to experimenter's mistake.

(B) Experimental error is required for comparing treatment effects.

(C) Experimental error is required for testing the significance of treatment effects.

(D) Experimental error is due to extraneous factors.

154. The maximum possible number of orthogonal contrasts among four treatments is :

(A) 1

(B) 2

(C) 3

(D) 4

155. In a completely randomized design with t treatments and n experimental units, error degrees of freedom equals:

(A) $n - t$

(B) $n - t - 1$

(C) $n - t + 1$

(D) $t - 1$

156. The dead bodies coming to a burial ground is an example of :

(A) Pure Birth Process

(B) Pure Death Process

(C) Birth and Death Process

(D) Constant rate of arrival

157. The system of loading and unloading of goods usually follows :

(A) LIFO (B) FIFO

(C) SIRO (D) SBP

158. The designation of Poisson arrival, exponential service, limited queue, selected randomly in a single server is represented by :

(A) $M / E / 1 : \infty / SIRO$

(B) $M / E / S : \infty / SIRO$

(C) $M / M / 1 : N / SIRO$

(D) $M / E / S : N / SIRO$

159. If expected value of an estimator is equal to the corresponding population parameter, the estimator is called :

(A) Efficient

(B) Unbiased

(C) Consistent

(D) Biased

160. Mean squared error corresponding to an estimator T equals :

(A) Variance of T + (Bias of T)^2

(B) Standard Deviation of T + (Bias of T)^2

(C) (Expectation of T)^2 + (Bias of T)^2

(D) Expectation of T + (Bias of T)^2

161. Neyman-Fisher factorization theorem is used to get :

(A) Efficient estimator

(B) Sufficient estimator

(C) Unbiased estimator

(D) Complete estimator

162. Suppose $X_1, X_2, \ldots , X_n$ is a random sample from $N(\mu, \sigma^2)$ population with μ and σ^2 unknown. Define $\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 = (n-1)s^2 = nS^2$ where $n\bar{X} = \sum_{i=1}^{n} X_i$. Which of the following is not correct?

(A) s^2 is an unbiased estimator of σ^2

(B) $\bar{X}$ is an unbiased estimator of μ

(C) S^2 is an unbiased estimator of σ^2

(D) S^2 is a maximum likelihood estimator of σ^2

163. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $U(-\theta, \theta)$ population. Which of the following is a sufficient statistic?

(A) $\min_{i \le i \le n} X_i$

(B) $\max_{i \le i \le n} X_i$

(C) $\max_{i \le i \le n} |X_i|$

(D) $(\min_{i \le i \le n} X_i, \max_{i \le i \le n} X_i)$

164. Let $X_1, X_2, \ldots, X_n$ be a random sample from a binomial (n, p) population with $p \in \{0.1, 0.2, 0.3, \ldots, 0.9, 1\}$. Define $T = \sum_{i=1}^{n} X_i$. Then T becomes complete if :

(A) n takes any positive integral value

(B) n takes any value such that $n < 10$

(C) n takes any value such that $n > 10$

(D) n takes the value 10

165. Which of the following is correct?

(A) Variance of an MVUE cannot be smaller than Cramer-Rao lower bound

(B) MVUE may not be unique

(C) Cramer-Rao lower bound gives a lower bound to the variance of any linear unbiased estimator

(D) There exists some estimator whose variance is smaller than Cramer-Rao lower bound

166. Let X_i, $i = 1, 2, \ldots, n$ be a random sample from $N_p(\mu, \Sigma)$, where $\Sigma = ((\sigma_{ij}))$. Define $A = \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})' = ((a_{ij}))$, say, where $\bar{X}$ is the sample mean. Then which of the following is correct?

(A) $E(a_{11}) = (n-1)\sigma_{11}$

(B) $E(a_{11}) = n\sigma_{11}$

(C) $E(a_{pp}) = np\sigma_{11}$

(D) $E(a_{pp}) = (n-1)p\sigma_{11}$

167. Suppose $\hat{Y}$, $\hat{X}$, respectively, denote conventional unbiased estimators of population total of the study variable y and the auxiliary variable x where X is known. In ratio method of estimation, $\hat{Y}$ is improved upon by :

(A) multiplying $X/\hat{Y}$

(B) multiplying $X/\hat{X}$

(C) multiplying $\hat{X}/\hat{Y}$

(D) multiplying $\hat{Y}/\hat{X}$

168. Suppose y and x, respectively, denote the study variable and the auxiliary variable. Ratio and Regression estimator of population total of the study variable become identical if :

(A) $y \propto x$

(B) $y = c + x$ for some non-zero c

(C) $y \not\propto x$

(D) y is much larger than x

169. Suppose N and n, respectively, denote the population size and the sample size. If $n = N$:

(A) SRSWR becomes at least as good as SRSWOR

(B) under SRSWR it may be a census

(C) under SRSWR it must be a census

(D) under SRSWR it must not be a census

170. Neyman's allocation results in a sub-sample size that :

(A) may vanish

(B) never requires more than 100 percent sampling

(C) may require more than 100 percent sampling

(D) is more than corresponding stratum mean

171. Identify the false statement :

(A) Sampling variance of linear systematic sampling sometimes becomes less than that of an equivalent simple random sampling

(B) Linear systematic samples form a partition of the population

(C) In case of linear systematic sampling, sample mean is an unbiased estimator of population mean

(D) In case of circular systematic sampling, sample mean is an unbiased estimator of population mean

172. Let X be a lifetime random variable having density function f and survival function $\bar{F}$. Which of the following is not correct?

(A) Failure rate of X is increasing if, for any $x > 0, \bar{F}(x+t)/\bar{F}(t)$ is decreasing in t

(B) Failure rate of X is decreasing if $\bar{F}(x+t)/\bar{F}(t)$ is decreasing in t

(C) If failure rate of X is a constant, then X must follow an exponential distribution

(D) Failure rate of X can be non-monotone

173. For a non-negative random variable X, which of the following is correct?

(A) The failure rate of X cannot be first increasing, then decreasing and then increasing

(B) The failure rate of X can never be 0

(C) If failure rate of X is increasing, then its failure rate can be made decreasing by changing the value of the parameter of the underlying distribution

(D) Exponential distribution cannot be considered as a linear failure rate distribution

174. Which of the following is correct?

(A) The life of a parallel system is less than that of a series system

(B) The failure rate of a parallel system is the sum of the component failure rates

(C) The failure rate of a series system is the sum of the component failure rates

(D) Weibull distribution has a non-monotone failure rate

175. Suppose X and Y are two normal random variables. Which of the following is not correct?

(A) A normal distribution is always symmetric

(B) Normal distribution is a mesokurtic distribution

(C) The distribution of $X + Y$ is normal

(D) Normal distribution has all order moments finite

Rough Work

Rough Work

Rough Work